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ON STATISTICAL DEFINITION OF FREE AND FAIR ELECTION: BIVARIATE NORMAL DISTRIBUTION MODEL

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ABSTRACT

The coining of the expression free and fair was a good way towards evaluating elections, but fell short of qualifying its real quantification to guide an informed judgment; this paper provides guidance for such a definition.

Data from the Uganda National Baseline Survey were used to assess the dynamics of the determinants for a free and fair election. All determinants were statistically significant ($p<0.01$) for the two multinomial models (free and fair election models). The predicted probabilities for free and fair were each used as inputs to form probability distribution function could jointly define the expression free and fair using a bivariate normal distribution. A strong positive correlation was identified between an election being free and fair ($\rho = 0.9693, p < 0.01$) implying the reliability of the statistical models in jointly considering free and fair.

The study recommends development of central statistical computational system to inform electoral bodies and judges in passing scientifically backed ruling on whether an election is free and fair. A threshold percentage for any election to be referred to as free and fair could be developed either deterministically or stochastically and provisions of which passed under electoral law.

Keywords: Probability, Multivariate analysis, Election, Bayesian methods.

Contribution/ Originality

The paper contributes the first logical analysis of a national governance household baseline survey data and consequently proposes a framework for defining free and fair election. Given that free and fair is considered jointly, the paper recommends a definition based on joint bivariate probability distribution function.

1. INTRODUCTION

Globally, over 95% of the countries conduct elections, about the same percentage of the elections do not often end without controversy and for each of them, the expression *free and fair* is applied. For example, the 1995 constitution of the republic of Uganda, in reference to the functions of the Electoral Commission (EC), under article 61(a) empowers the EC “to ensure that regular, free and fair elections are held.” [1-3]

Free and fair (FnF) expression is used internationally to mean an ideal and excellent conduct of an election, a means by which citizens exercise their democratic right to vote leaders of their choice in a representative democracy [4-7]. For purposes of creativity, some election observers add another word *peaceful* to read *free, fair and peaceful* especially when referring to those elections conducted in developing countries, specifically in Africa [8].

However, the creativity instead generates a replication in the sense that *free* in FnF means allowed, permitted, able, welcome, unrestricted, at liberty and open while *fair* means reasonable, just, open-minded, impartial, rational and unbiased among other synonyms that already imply *peaceful*.

Literally, the superlative in the chronology of *fair* is *excellent* that means outstanding, brilliant, exceptional, first-rate, admirable, superb and tremendous among others. The coining of the expression *free and fair* probably implied a straight concession that the ideal election is not achievable under any practical criteria on the planet earth.

Nonetheless, neither the architects nor the users of the expression FnF were explicit about the confidence limits when they refer to an election being free and fair. This ambiguity in the use of the expression FnF while passing judgment about the conduct of an election often raises a number of unanswered questions: What are the indicators used to assess FnF? What threshold proportion of FnF is applied?

And the most critical one being, what is the margin of error (MoE) employed? [9] None of these questions are benchmarked even in the courts of law to rule that an election was free and fair or otherwise. It all lies in the proposed definition at a time which is always largely subjective. The objective of this paper is to present a framework for a statistical definition of free and fair in its current context without necessarily conceiving another or a better expression.

In statistical terms, margin of error is the maximum likely difference between the point estimate of a parameter and the actual value of the parameter, sometimes referred to as the maximum error of the estimate.

Margin of error is inversely proportional to the confidence level (*commonly 90%, 95% and 99%*), and directly proportional to the level of significance of the estimate being conducted. It guides one towards making an informed decision or judgement as to whether, say an election is free and fair and what would happen if say adjustments are made between significance levels and the parameter being estimated.

This paper presents multivariate statistical models that evaluate the dynamics of determinants for a free election as well as a fair election; computes the probabilities for each and subsequently performs stochastic transformations to propose a framework of a statistical definition for a *free and fair* election.

2. METHODOLOGY AND DATA

2.1. Data Sources and Description

The data used in this study were derived from the Uganda National Governance Baseline Survey (UNGBS) conducted by the Uganda Bureau of Statistics in collaboration with Makerere University, School of Statistics and Planning.

Table-1. Description of model variables

Number	Variable	Variable Description
1	urbanrural	urban or rural residence
2	region	Region
3	age	age of respondent
4	hhhead	head of household
5	mstatus	marital status

Continue

6	readwrite	can read and write
7	attendschool	ever attended school
8	employstatus	employment status
9	dwelling	dwelling unit decency status
10	ownhouse	house ownership status
11	watersource	source of water
12	lighting	source of lighting
13	ownland	land ownership status
14	granary	granary presence
15	sex	sex of respondent
16	disability	disability status
17	meals	meals per day
18	free_hybrid	Free election
19	fair_hybrid	Fair election

A national sample of 4776 households was scientifically drawn and data collected on various themes of governance among which was the theme on electoral system. Table 1 show nineteen variables that were carefully chosen for this study whereof the last two variables, namely; *free_hybrid* and *fair_hybrid* were each a dependent variable in the models developed.

These variables were developed after a thorough cross linkage with other variables of interest; in a way building more reliable dependent variables by performing different scenario checks [10]. In most developing countries and largely, many types of electoral irregularities are known and prominent, but this study considered only those that are eminent and were collected in the governance baseline survey.

2.2. Methodology for Free and Fair Dynamics

A conceptual framework was developed to show the relationship that exists between the independent variables and the hybrid dependent variables for the study. The two hybrid dependent variables were derived as explained in subsection 2.1, but with the same determinants and intervening variables as shown in Figure 1.

The variable *free_hybrid* was developed based on whether the respondent checked the register, received offers to vote or witnessed vote counting. We note that, in Uganda, the constitution, under article 68(3) states that a candidate is entitled to be present in person or through his or her representatives or polling agents at the polling station throughout the period of voting, counting of the votes and ascertaining the results of the poll. [1, 11].

The construction of the dependent variable *fair_hybrid* was based on whether; the respondent found when the register showed that he/she had voted already, was pressured to vote, media gave equal opportunities to candidates and whether the respondent witnessed any irregularities. The independent variables were primarily characteristics for the typical citizens, intervened by characteristics for location and services.

Figure-1. Conceptual framework for determinants of free and fair elections

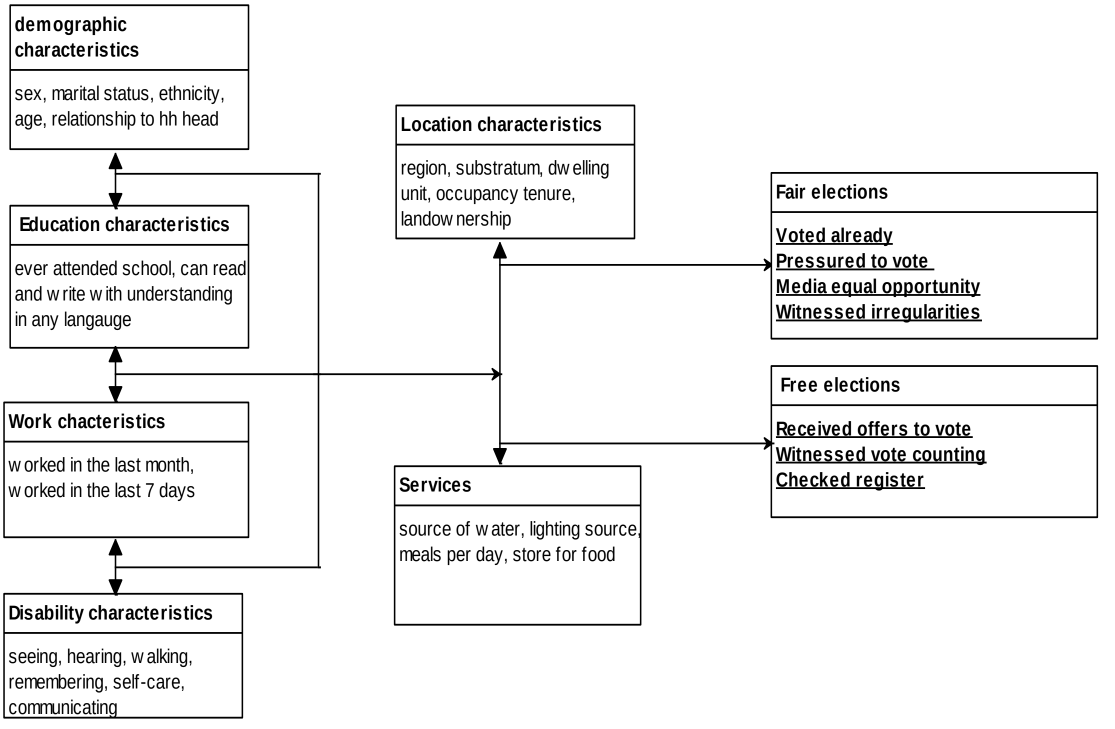
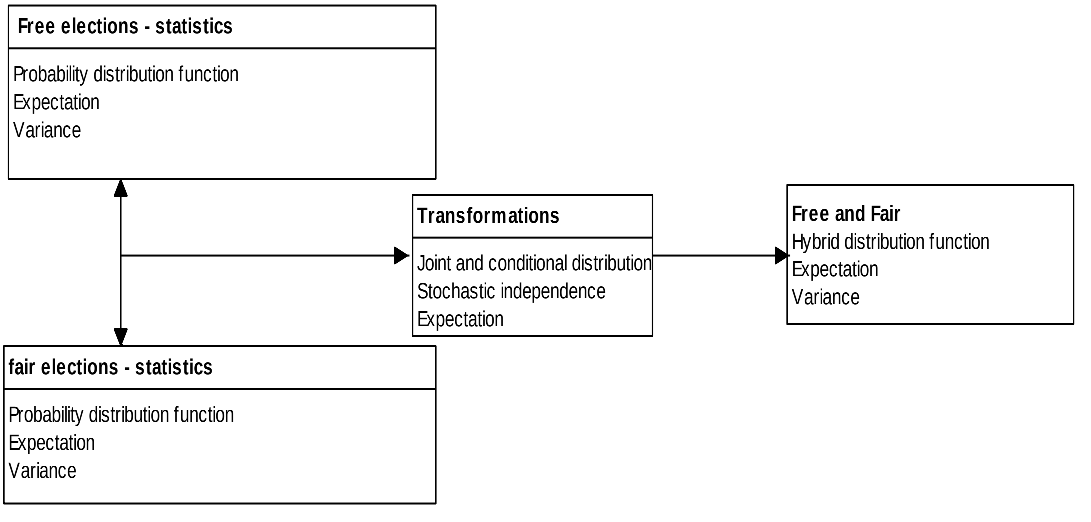


Figure-2. Stochastic framework for free and fair elections



3. FINDINGS

3.1. Introduction

Findings are presented in such an order that begins with the two multinomial models and their tests (Table 2) followed by histograms for derived post estimation probabilities of free and fair (Figure 3).

3.2. Model Representation for Determinants of Free and Fair Election

Two multinomial models were developed each for estimating *free* and *fair* with the base category *'sometimes'* so as to evaluate predictors for *'free'* and *'fair'* respectively. All variables presented were statistically significant predictors as shown in Table 2. There were a number of commonalities in the exponentiated coefficients with most of them revealing a positive increase in the predicted variable given a unit change in the predictors. However, the following variables revealed a reduction in the levels of the predicted variables for any unit change in their coefficients; residence category (*rural/urban*), ability to read and write, disability status, decency of the dwelling unit, source lighting and water. For example, the respondents with private sources of lighting and water were more likely to assess the election as being not free and fair than their counterparts who had access to public sources of lighting and water. Similarly respondents residing in rural areas were about 15% less likely to assess elections favourably than their counterparts residing in urban areas. Model tests and comparison using log-likelihood, chi-square, AIC and BIC compared to the singular models for free and fair resulted in the selection of the models presented in Table 2 as being the best possible. It is also of interest to know that all standard errors for coefficients were small enough to support the reliability of the multinomial models developed for free and fair election [12].

Table-2. Models for dynamics of predictors for free and fair election

Multinomial Logistic Regression Models for Free and Fair Elections				
Dep. vars: *free, fair and free and fair;* **base:** *sometimes*	Model One: Free elections		Model Two: Fair elections	
	Not Free	Free	Not Fair	Fair
Urban or Rural residence				
Urban	1.000	1.000	1.000	1.000
Rural	0.887** (-0.001)	0.843** (-0.002)	0.884** (-0.002)	0.955** (-0.003)
Sex of respondent				
Male	1.000	1.000	1.000	1.000
Female	1.135** (-0.003)	0.557** (-0.002)	1.006+ (-0.003)	0.909** (-0.005)
Head of household				
not head	1.000	1.000	1.000	1.000
Head	1.999** (-0.006)	1.601** (-0.007)	1.672** (-0.005)	1.060** (-0.007)
Marital status				
Single	1.000	1.000	1.000	1.000
Married	1.704** (-0.003)	1.636** (-0.004)	1.693** (-0.003)	1.083** (-0.004)
Ever attended school				
never attended school	1.000	1.000	1.000	1.000
attended school	1.011* (-0.005)	1.589** (-0.01)	0.794** (-0.004)	1.312** (-0.014)
Can read and write				
unable to read and write	1.000	1.000	1.000	1.000

Continue

Multinomial Logistic Regression Models for Free and Fair Elections				
Dep. vars: *free, fair and free and fair;* **base:** *sometimes*	Model One: Free elections		Model Two: Fair elections	
	Not Free	Free	Not Fair	Fair
able to read at least	0.785** (-0.003)	0.926** (-0.005)	1.131** (-0.004)	1.812** (-0.014)
Employment status				
not employed	1.000	1.000	1.000	1.000
Employed	2.245** (-0.006)	2.506** (-0.01)	1.841** (-0.005)	0.698** (-0.004)
Disability status				
Disabled	1.000	1.000	1.000	1.000
Not disabled	0.612** (-0.002)	0.528** (-0.002)	0.597** (-0.002)	0.423** (-0.002)
Age	1.069** (0)	1.050** (0)	1.064** (0)	1.007** (0)
Meals per day status				
less than three meals	1.000	1.000	1.000	1.000
three or more meals	0.985** (-0.003)	1.247** (-0.005)	1.096** (-0.003)	1.512** (-0.008)
Dwelling unit decency status				
not decent	1.000	1.000	1.000	1.000
decent dwelling	0.528** (-0.001)	0.501** (-0.002)	0.510** (-0.002)	0.691** (-0.004)
House ownership status				
not own/free	1.000	1.000	1.000	1.000
own/free	1.407** (-0.006)	1.203** (-0.007)	1.247** (-0.005)	1.288** (-0.011)
Source of water				
Public	1.000	1.000	1.000	1.000
Private	0.973** (-0.002)	0.960** (-0.002)	0.970** (-0.002)	1.239** (-0.004)
Source of lighting				
Public	1.000	1.000	1.000	1.000
Private	0.833** (-0.001)	0.883** (-0.002)	0.859** (-0.001)	1.066** (-0.003)
Land ownership status				
do not own land	1.000	1.000	1.000	1.000
own land	1.145** (-0.004)	1.524** (-0.007)	1.344** (-0.004)	1.152** (-0.008)
Granary availability				
no granary	1.000	1.000	1.000	1.000
has granary	1.142** (-0.003)	1.171** (-0.005)	1.341** (-0.004)	1.660** (-0.009)

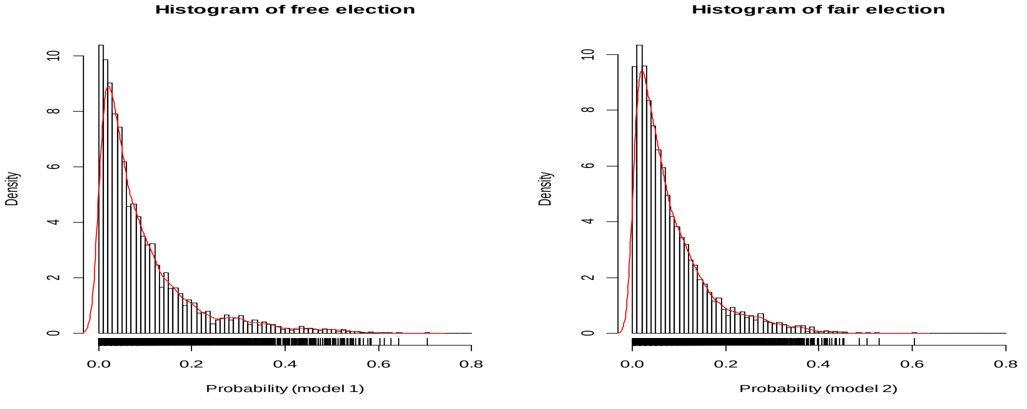
Note: Exponentiated coefficients; Standard errors in parentheses and (+ means p<0.10, * means p<0.05, ** means p<0.01)

No. of households	4771	4771
Log-likelihood	-4.65E+06	-3.14E+06
χ^2	1107954	9.25E+05
Df	32	32
AIC	9.29E+06	6.29E+06
BIC	9.29E+06	6.29E+06

Post-estimation computations were carried out to derive the probabilities from the multinomial models for free and fair respectively. Figure 3 shows the details and it can clearly be observed that these probabilities are all positively skewed with averages of less than two percent. This implies that the models used to independently predict free and fair have a high level of reliability given that there was no significant variability between probabilities in the two independently developed models.

3.3. Descriptive Statistics for the Probability of Free and Fair Election

Figure-3. Graphical analysis of model derived probabilities for free, fair and free and fair election



4. DISCUSSIONS

We approach our discussions by modelling the household or respondent based probabilities presented in section 3 as random variables for *free and fair*. We note that judges in the constitutional courts of judicature or otherwise are often confronted with the challenge of deciding whether an election is or is not *free and fair*. Our case, presents a two-variable scenario, hence the discussions employed the bivariate joint distribution function.

4.1. Statistical Definition of Free and Fair from First Principles

Household based probability estimates from the multivariate statistical analysis for free and fair predictions (Table 2) were applied to form a joint probability distribution function that defined free and fair. Since the estimates constitute continuous random variables, we opted for the joint probability density functions for the continuous bivariate case [13]. We present some definitions to guide the logical development of the definition for free and fair. We assume that the multivariate model predictions are based on two random variables F_1 and F_2 representing free and fair respectively.

Therefore, the joint probability distribution of F_1 and F_2 is given by a nonnegative function, election is free and fair, $\mathrm{e}(\mathrm{f}_1, \mathrm{f}_1)$ such that:

$$P(a_1 \leq F_1 \leq a_2,\ b_1 \leq F_2 \leq b_2) = \int_{b_1}^{b_2} \int_{a_1}^{a_2} e(f_1, f_2) df_1 df_2 \qquad (1)$$

While the marginal probability density functions of F_1 and F_2 are given by

$$e_1(f_1) = \int_{-\infty}^{\infty} e(f_1, f_2) df_2 \qquad (2)$$

and

$$e_2(f_2) = \int_{-\infty}^{\infty} e(f_1, f_2) df_1 \quad (3)$$

However, in making a final ruling, an informed judge would be interested in the conditional probability density functions of F_1 given $F_2 = f_2$, as defined by election is *free* given it is *fair[13, 14]*:

$$e(f_1|f_2) = \begin{cases} \frac{e(f_1,f_2)}{e_2(f_2)} & \forall\ e_2(f_2)>0 \\ 0 & elsewhere \end{cases} \quad (4)$$

Similarly, the conditional probability density function of F_2 given $F_1 = f_1$, defined by election is fair given it is free:

$$e(f_2|f_1) = \begin{cases} \frac{e(f_1,f_2)}{e_1(f_1)} & \forall\ e_1(f_1)>0 \\ 0 & elsewhere \end{cases} \quad (5)$$

The questions of whether an election would be fair and stochastically independent of free or free and stochastically independent of fair may probably not arise when referring to a free and fair election because a fair election cannot be independent of a free election. However, in case such a scenario arises, its statistical definition would easily be interpreted as:

$$e(e_1, e_2) = e_1(f_1) \times e_2(f_2) \qquad \forall\ f_1, f_2 > 0 \quad (6)$$

To guide the final judgement, a common statistical measure; the mean, was be applied. Currently, the totality of historical perspective of the election period is hypothetically reflected without significantly reliable evidence, thus leaving some aggrieved parties always unsatisfied with the decision. Accordingly, the expected value of the function $g(F_1, F_2)$ defined as the sum over all values of (f_1, f_2) for which $p(f_1, f_2) > 0$ is applied. Thus, given that (F_1, F_2) are continuous random variables, with probability $e(f_1, f_2)$, then:

$$E[g(F_1, F_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(f_1, f_2)\, e(f_1, f_2) df_1 df_2 \quad (7)$$

It might be of interest to establish the covariance between '*being free*' and '*being fair*' since the two seem to be more dependent than independent occurrences. This led to yet another definition for covariance between two random variables F_1 and F_2 defined as:

$$cov(\, F_1, F_2\,) = E[\, (F_1 - \mu_1)(F_2 - \mu_2)\,] \quad (8)$$

or

$$cov(\, F_1, F_2\,) = E[F_1 F_2] - \mu_1 \mu_2 \quad (9)$$

Where;

$$\mu_1 = E[F_1];\ \mu_2 = E[F_2] \quad (10)$$

In our definition of free and fair, we applied the notion that if F_2 tends to be large when F_1 is large, and if F_2 tends to be small when F_1 is small, then a conclusion is arrived at that F_1 and F_2 have a positive covariance. On the other hand, if F_2 tends to be small when F_1 is large, and if F_2 tends to be large when F_1 is small, then F_1 and F_2 have a negative covariance. While covariance will enable us measure the direction of the association between F_1 and F_2 , correlation measures the strength of the association and for this case, and is defined as:

$$\rho = \frac{cov(F_1, F_2)}{V(F_1) \times V(F_2)} \quad (11)$$

It should be recalled that the correlation coefficient is a unit-less quantity that takes on values between -1 and +1. If $\rho = \pm 1$, then F_2 must be a linear function of F_1.

4.2. Applying the National Governance Baseline Data on Free and Fair Definition

Following the discussions in subsection 4.1, we assume a bivariate normal distribution and present the definition for *free* and *fair* with the joint probability distribution as below [15-17]:

$$p(f_1, f_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} exp\left[-\frac{Z}{2(1-\rho^2)}\right] \quad (12)$$

where

$$Z = \frac{(f_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(f_1-\mu_1)(f_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(f_2-\mu_2)^2}{\sigma_2^2} \quad (13)$$

and

$$\rho \equiv Cor(f_1, f_2) = \frac{V_{12}}{\sigma_1\sigma_2} \quad (14)$$

The above *free* and *fair* definition has, at minimum the following properties:

$$\int_{f_2}\int_{f_1} e(f_1,f_2)df_1df_2=1 \qquad e(f_1,f_2)\geq 0 \quad (15)$$

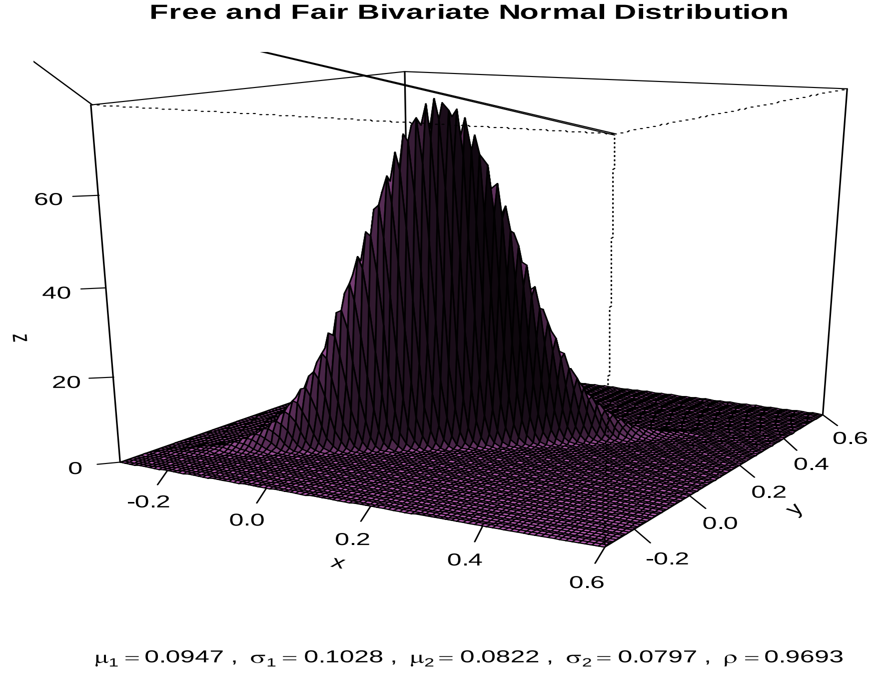
From the national governance baseline data, the following analyses were made;

Table-3. Descriptive statistics for the marginal probabilities of free and fair

Statistics	**Free (f_1)**	**Fair (f_2)**
Mean	0.0947	0.0822
Standard Error	0.0015	0.0012
Median	0.0588	0.0569
Mode	0.0920	0.0949
Standard Deviation	0.1028	0.0797
Sample Variance	0.0106	0.0064
Kurtosis	4.4362	3.4094
Skewness	2.0043	1.7389
Range	0.7057	0.6033
Minimum	0.0004	0.0005
Maximum	0.7062	0.6038
Sum	452.2697	392.5224
Count	4776	4776
Correlation coefficient	0.9693	

Furthermore, using findings in Table 3 and coding the bivariate normal distribution by applying R language for statistical computing for the free and fair definition as stated above, a surface in the $X - Y$ plane relating (f_1, f_2) was generated as shown in Figure 3. It should be noted that the individual 1-dimensional distributions for (f_1) and (f_2) also presented approximately normal distribution functions [18-20].

Figure-4. Bivariate normal plot for definition of free and fair



Therefore, for any judgement to be passed on whether an election is *free* and *fair*, it should be based on an acceptable level of probability which should then be documented in the laws of the country that practices democratic principles through elections. The joint probability can be computed using the following expression and the integration done numerically [21]:

$$P(a_1 \leq f_1 \leq a_2 \cap b_1 \leq f_2 \leq b_2) = \int_{b_1}^{b_2} \int_{a_1}^{a_2} e(f_1, f_2) df_1 df_2 \quad (16)$$

It can be seen that for normalised variables $z_{f_1} = (f_1 - \mu_{f_1})/\sigma_{f_1}$ and $z_{f_2} = (f_2 - \mu_{f_2})/\sigma_{f_2}$, the bivariate normal probability distribution function becomes:

$$e(f_1, f_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} exp\left[-\frac{Z_{f_1}^2 + Z_{f_2}^2 - 2\rho z_{f_1} z_{f_2}}{2(1-\rho^2)}\right] \quad (17)$$

The only parameter in this case that would determine the variation in the bivariate standard normal distribution is ρ, the correlation between *free* (f_1) and *fair* $f_2)$ [19, 22]. If $\rho = 0$, then *free* (f_1) and *fair* (f_2) are independent, implying that there is no need for them to be combined in the same expression that reads "free and fair". As ρ increases, the distribution is stretched diagonally, forming elliptical isopleths with positive sloped major axes which are ideal for the definition of *free* and *fair* elections (Figure 4). A more justification for the definition of the expression free and fair is appreciated when two conditional probabilities *free* given *fair* and *fair* given *free* are examined. However, given that *fair* is more conclusive than *free* and the parent distribution is a bivariate normal, we shall define the conditional probability for a *fair* (f_2) election given it is *free* (f_1) as follows [14, 21, 23, 24]:

$$e((f_2|f_1) \sim N(\mu_{(f_2|f_1)}, \sigma^2_{(f_2|f_1)}) \quad (18)$$

Where

$$\mu_{(f_2|f_1)} = \mu_{f_2} + \rho\sigma_{f_2} \, {(f_1 - \mu_{f_1})}\big/{\sigma_{f_1}} \tag{19}$$

and

$$\sigma_{(f_2|f_1)} = \sigma_{f_2}\sqrt{1-\rho^2} \tag{20}$$

5. CONCLUSIONS

This paper contributes to the most contentious human rights phenomenon in the world of whether in taking some decision, one is free and fair. Specifically, it addresses the more fundamental perspective of governance when the electorates decide their leaders through casting votes as they exercise their democratic right to make the world a better place for all humanity. Given the participatory nature of the process, many indicators have to be assessed to provide scientifically acceptable level of confidence in the outcome of an election. To achieve that, a standard definition has to exist to provide a backup of the final decision of the outcome of an election. Relatedly, some standards have to be developed for instance, the fifty plus percentage minimum for a presidential candidate to be declared winner, otherwise such an election is repeated, a popular practice in many countries where elections are held.

However, a more important characteristic of what percentage should be considered for judging an election as free and fair still remains and has had tremendous costs in politics. Some facts, though still remain known that; no election can be 100% free and fair, but it has been maintained that despite the inadequacies, there is always a winner [25, 26]. The principle of relativity in judging whether an election is free and fair, whereby if candidate A is leading candidate B with a certain margin, (A-B), but with a big error, E of lots and loads of inconsistencies is never applied. Instead the practice is often to declare candidate A as a winner. Assume the null hypotheses $H_0: \mu = FnF(\textit{election is free and fair})$ against the alternative $H_0: \mu \neq FnF$ (*not free and fair*). If we failed to reject H_0 when actually the election is not free and fair, in statistical terms, we commit Type I error which is the worst error known to be committed [8, 27]. Would this error be minimised? [28, 29] To minimum this error, we employ scientific definitions such as the one presented in this paper; develop means of data collection on key electoral outcome indicators; develop central statistical computational system that will contain the standardised definitions, indicators, data and the ability for effective and timely generation of the necessary statistical information to guide judges in passing scientifically backed ruling about an election's being free and fair. A threshold level of the minimum percentage for any election to be referred to as free and fair, is recommended for development either by deterministic or stochastic provisions passed as an electoral law [28]. Such a system, as recommended, offers a more reliable source of decision making, cannot be questioned because it is based on evidence and as such would minimise misunderstandings and possible loss of lives. A free and fair election can be defined as described in Equations (16) through (20).

6. ACKNOWLEDGEMENTS

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